A Transportation Network Design Model with Equity Constraints

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Transportation Network Design is based on simulation models that allow to define supply and activity systems that best satisfy the prefixed objectives.

The Network Design Problem (NDP) can be classified into two types:

**DISCRETE NETWORK DESIGN PROBLEM**

determines an optimal set of locations for constructing some new facilities added to a current transportation network

**CONTINUOUS NETWORK DESIGN PROBLEM**

determines the optimal enhancements for some existing facilities.

In literature different models for solving NDP assuming single or multiple objective functions and solution algorithms have been presented.
Equity can be defined along many facets such as justice, rights, treatment of equals, capability, opportunities, resources and so on.

There are three major categories of transportation equity:

**HORIZONTAL EQUITY**

distribution of impacts between individuals and groups considered equal in ability and need; according to this definition, equal individuals and groups should receive equal shares of resources, bear equal costs, and in other ways be treated the same.

**VERTICAL EQUITY with regard to income and social class**
distribution of impacts between individuals and groups that differ by income or social class; This definition is used to support affordable modes, discounts and special services for economically and socially disadvantaged groups (social inclusion).

**VERTICAL EQUITY with regard to mobility need and ability**
distribution of impacts between individuals and groups that differ in mobility ability and need; transport facilities and services accommodate all users, including those with special needs.
In transportation systems **unreliability and incompleteness** of available information and **uncertainty** due to user behaviour and manager knowledge are evident.

For example:

**Uncertainty on LINK CAPACITY**  
estimated/actual road link capacity is an uncertain variable and then the values from link cost functions too.

**Uncertainty on DEMAND LEVEL**  
could be known in approximate and/or uncertain way.

**Uncertainty on AVAILABLE BUDGET**  
could be known in approximate and/or uncertain way.

**Uncertainty on network PERFORMANCE INDICATORS**  
traffic managers could aim at improving some network performance indicators on a set of road links or on a specific route with respect to the current situation (how much? slightly, medium, heavily? ).
A considerable amount of research effort has been devoted to road Network Design Models over the last forty years.

Few works have focused the problem of including Uncertainty within the Network Design Problem.

Das et al., 1999; Mudchanatongsuk et al., 2008; Selim and Ozkarahan, 2008; Ghatee and Hashemi, 2009a and 2009b; Caggiani and Ottomanelli, 2011; 2012.

Even less authors have considered also Equity Constraints in the Network Design Problem.


In order to consider both Uncertainty and Equity in Network Design Problem, we propose a new problem formulation that has been specified as a fixed point fuzzy optimization problem subjected to a set of flexible constraints.
In the classic CONTINUOUS NDP, the optimal network enhancements are determined by minimizing the total system cost under a set of constraints.

However, the equilibrium Origin-Destination (O-D) travel costs between some O-D pairs may be increased or decreased after implementing an optimal network design scenario, leading to positive or negative results for network users.

**Example Network**

**O-D travel costs BEFORE total network costs optimization**

\[ C_{1-4} = 3.00 \quad C_{2-4} = 3.25 \]

**O-D travel costs AFTER total network costs optimization**

\[ C^*_{1-4} = 2.10 \quad C^*_{2-4} = 3.65 \]

**EQUITY PERFORMANCE INDICATOR** (cost/benefit degree of equitability distribution)

\[ \alpha_{1-4} = \frac{C^*_{1-4}}{C_{1-4}} = 0.70 \quad \alpha_{2-4} = \frac{C^*_{2-4}}{C_{2-4}} = 1.12 \]

\[ \alpha_{\text{MAX}} = 1.12 \]
The classic **CONTINUOUS NDP with EQUITY CONSTRAINTS** the optimization problem can be specified as follows:

\[
\mathbf{x}^* = \arg\max_{\mathbf{x}} w(\mathbf{x}, \mathbf{f}^*)
\]

subject to:

\[
\mathbf{f}^* = (\Delta(\mathbf{x}) \mathbf{P}(\mathbf{x}, \mathbf{C}(\mathbf{x}, \mathbf{f}^*))) \mathbf{d}(\mathbf{C}(\mathbf{x}, \mathbf{f}^*))
\]

\[
x, \mathbf{f}^* \in Eq \begin{cases}
\alpha_{\text{MAX}} = \max_{od \in OD} \left\{ \frac{C_{od}^* (\mathbf{x}, \mathbf{f}^*)}{C_{od} (\mathbf{x}, \mathbf{f}^*)} \right\} \\
\alpha_{\text{MAX}} \leq \beta_{\text{MAX}}
\end{cases}
\]

\[
x \text{ is the vector of the design variables} \\
w \text{ is the objective function} \\
f^* \text{ is the vector of traffic flows} \\
\Delta \text{ is the link-path incidence matrix;} \\
P \text{ is the path choice probability matrix;} \\
C \text{ is the vector of path costs;} \\
d \text{ is the vector of travel demand;}
\]

**CONSISTENCY CONSTRAINT**

among demand, flows and supply parameters

**EQUITY CONSTRAINT**

If \( \beta_{\text{MAX}} < 1 \) all users can benefit from the network design implementation, if \( \beta_{\text{MAX}} > 1 \) there will exist users who suffer a travel cost increase induced by the design implementation.
Classic optimization problems assume **FIXED RIGID VALUE OF EQUITY THRESHOLDS** (i.e. $\beta_{\text{MAX}}$). The equity thresholds are set by decision makers reflecting the community needs. Actually these thresholds are the results of approximate reasoning of a human expert whose knowledge is based on **incomplete and/or unreliable** of available data.

It would be more appropriate to consider as Equity Constraint of the NDP expressions like the following:

“The EQUITY PERFORMANCE INDICATOR should be as much as possible lower then the flexible/approximate threshold $\beta_{\text{MAX}}$”

\[ \alpha_{\text{MAX}} \leq \beta_{\text{max}} \]

In our model we propose to specify the thresholds as flexible values and represented by **FUZZY SETS**.
STATEMENT OF THE PROBLEM: FUZZY CONSTRAINTS

**A possible interpretation of fuzzy Equity Constraint**

"\( \alpha_{\text{max}} \) approximately lower or equal to \( \beta_{\text{max}} \)"

**Objective function (total network cost minimization) considered as a fuzzy set**

"\( C_T \) approximately lower or equal to \( \beta_{\text{max}} \)"

Fuzzy numbers are represented as function of the degree of membership called **SATISFACTION (h)**. The closer to one the satisfaction is, the more the constraints are fulfilled.

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The optimization problem, considering the network total cost minimization as objective function, is transformed into the following FUZZY OPTIMIZATION PROBLEM:

\[
\max h \\
\text{subject to:}
\]

\[
C(x(h), f^*) \leq C_{T \max} \cdot (1 - h)
\]

\[
f^* = (\Delta(x(h))P(x(h), C(x(h), f^*))d(C(x(h), f^*))
\]

\[
\alpha_{\text{MAX}} \leq \beta_{\text{min}} + (\beta_{\text{max}} - \beta_{\text{min}}) \cdot (1-h)
\]

The objective function to be maximized is the satisfaction \( h \), and the value of optimization function becomes a further constraint to the problem (see for ex. Teodorovic and Vukadinovic, 1998).

**NETWORK TOTAL COST MINIMIZATION**

triangular membership function

**CONSISTENCY CONSTRAINT**

**FUZZY EQUITY CONSTRAINT**

trapezoidal membership function

The closer to one \( h \) is (maximization of satisfaction), the more the network total cost minimization and the constraints are satisfied.
The chosen approach to the problem is the global **OPTIMIZATION OF SIGNAL SETTINGS** that consists in determine the optimal effective green time for all intersections (vector $g^*$).

The numerical application has been carried out with the network and starting data considered by Yang H. et al. (2001).

### Test Network

![Test Network Diagram](image)

**O-D Matrix**

<table>
<thead>
<tr>
<th></th>
<th>6</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>120</td>
<td>150</td>
<td>100</td>
</tr>
<tr>
<td>2</td>
<td>130</td>
<td>200</td>
<td>90</td>
</tr>
<tr>
<td>4</td>
<td>80</td>
<td>180</td>
<td>110</td>
</tr>
</tbody>
</table>

**Waiting Time**

- Doherty Delay Function

**Travel Cost**

- BPR Link Cost Function
The fuzzy optimization problem of the numerical application is specified as follows:

\[
\text{max } h
\]

subject to:

\[
C(g(h), f^*) \leq C_{T\text{max}} \cdot (1 - h)
\]

\[
f^* = (\Delta(g(h))P(g(h), C(g(h), f^*))) \cdot d(C(g(h), f^*))
\]

\[
\alpha_{\text{MAX}} \leq \beta_{\text{min}} + (\beta_{\text{max}} - \beta_{\text{min}}) \cdot (1 - h)
\]

\[
f_{1-2} \leq 0.5 \cdot f^*_{1-2} \quad f_{1-4} \leq 0.5 \cdot f^*_{1-4}
\]

\[
\sum_{ph} g_{ph}^{nd} = \text{CycleTime}_{nd} \quad \forall \ nd \in \{5,6,8\}
\]
In order to verify the performance of the proposed model, starting from the same green time setting (SC) we have carried out **THREE DIFFERENT OPTIMIZATION PROBLEM** described as follows.

**CRISP OPTIMIZATION - (CO)**
The CO is the **CLASSIC CONTINUOUS NDP** with the minimization of network total cost. The only other requirements to be fulfilled are the crisp **vertical equity constraints** on the two links. It has been calculated the value of the **EQUITY PERFORMANCE INDICATOR** $\alpha_{\text{MAX}}$ and the value of **NETWORK TOTAL COST** (denoted by $C_{\text{T MAX}}$).

**EQUITY CRISP OPTIMIZATION - (ECO)**
The ECO is a **CLASSIC EQUITY BASED OPTIMIZATION** model that does not consider uncertainty in the equity constraints. The **horizontal equity constraint** has been added. We assume that decision makers have been fixed $\beta_{\text{MAX}} = 0.9\alpha_{\text{MAX}}$.

**EQUITY FUZZY OPTIMIZATION - (EFO)**
The EFO is based on the **PROPOSED MODEL**. The equity constraint of this problem is the same as the one considered in ECO but with uncertainty in its definition. In other word this equity constraint translates the following expression stated by decision makers: ”The maximum value of the equity performance indicator must be approximately lower than or equal to $0.9\alpha_{\text{MAX}}$".
The results achieved are summarized in the following table:

<table>
<thead>
<tr>
<th></th>
<th>$g_1^5$[s]</th>
<th>$g_2^5$[s]</th>
<th>$g_3^5$[s]</th>
<th>$g_1^6$[s]</th>
<th>$g_2^6$[s]</th>
<th>$g_1^8$[s]</th>
<th>$g_2^8$[s]</th>
<th>$\alpha_{\text{MAX}}$</th>
<th>$C_T$</th>
<th>$f^*_{1-2}$</th>
<th>$f^*_{1-4}$</th>
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<tbody>
<tr>
<td>SC</td>
<td>35</td>
<td>20</td>
<td>35</td>
<td>80</td>
<td>10</td>
<td>10</td>
<td>80</td>
<td>-</td>
<td>5333852</td>
<td>120</td>
<td>150</td>
</tr>
<tr>
<td>CO</td>
<td>54.3</td>
<td>23.1</td>
<td>12.6</td>
<td>53.6</td>
<td>36.4</td>
<td>34.3</td>
<td>55.7</td>
<td>10.2</td>
<td>3254904</td>
<td>59</td>
<td>75</td>
</tr>
<tr>
<td>ECO</td>
<td>55.8</td>
<td>21.8</td>
<td>12.4</td>
<td>57.6</td>
<td>32.4</td>
<td>33.2</td>
<td>56.8</td>
<td>9.15</td>
<td>3116790</td>
<td>60</td>
<td>75</td>
</tr>
<tr>
<td>EFO</td>
<td>60.8</td>
<td>15.2</td>
<td>14.0</td>
<td>62.6</td>
<td>27.4</td>
<td>30.9</td>
<td>59.1</td>
<td>6.68</td>
<td>3826857</td>
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<td>74</td>
</tr>
</tbody>
</table>

In the network total cost minimization the ECO has a better behaviour than the EFO ($C_{T_{\text{ECO}}} < C_{T_{\text{EFO}}}$). Fuzzy optimization presents an $\alpha_{\text{MAX}}$ value lower than that obtained through ECO; $\alpha_{\text{EFO}}$ is about 65% of $\alpha_{\text{CO}}$. In other words, in the EFO, the most disadvantaged O-D couple after the total cost optimization has a cost increment much lower than in the ECO case.
CONCLUSIONS

This research face the problem of defining quantitative design tools to deal with equity in transportation planning. In this work we proposed a formulation for including equity issues in network design problem.

We suggest to consider also flexible equity constraints explicitly represented by fuzzy sets. Then the proposed equity network design problem is then specified as a fixed point fuzzy programming problem.

The numerical analysis has shown the ability of the method to take into account conflicting constraints as in multi-objective optimization.

The proposed approach allows to better spread the disadvantages among all network users maintaining the highest increase of O-D pair costs (due to the global optimization) to lower values than the classic method.

We are finalizing the application of the method to a real sized network were the first results are similar to the ones obtained on the current network.

Further research activities are dealing with a formulation of the model for the transit network design problem.
THANK YOU
FOR YOUR ATTENTION